Empirical Mode Decomposition[1] for High Density, Dipole-Confined Plasmas

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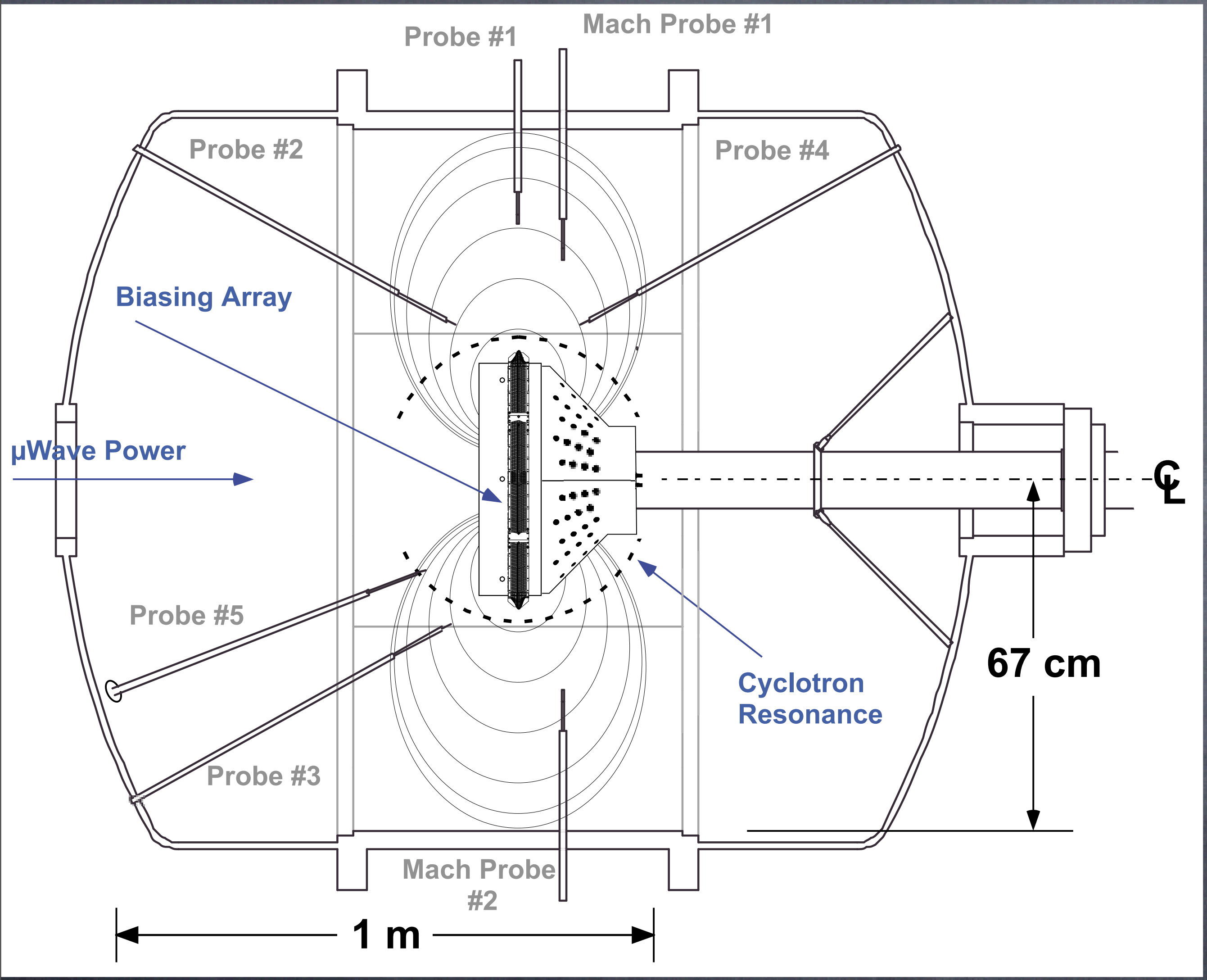
Dipole Density Regimes

- A dipole-confined plasma has two distinct density regimes, namely low and high density.
- Low density: $p_n < 10^{-5}$ Torr
- High Density: $p_n \geq 10^{-5.3}$ Torr
- The High Density regime displays low frequency (3-8kHz) turbulence.



CTX Dipole

B_{max}~2kG, B_{wall}~50G

$L_{Terella}$ =20cm

$L_{Chamber}$ =70cm

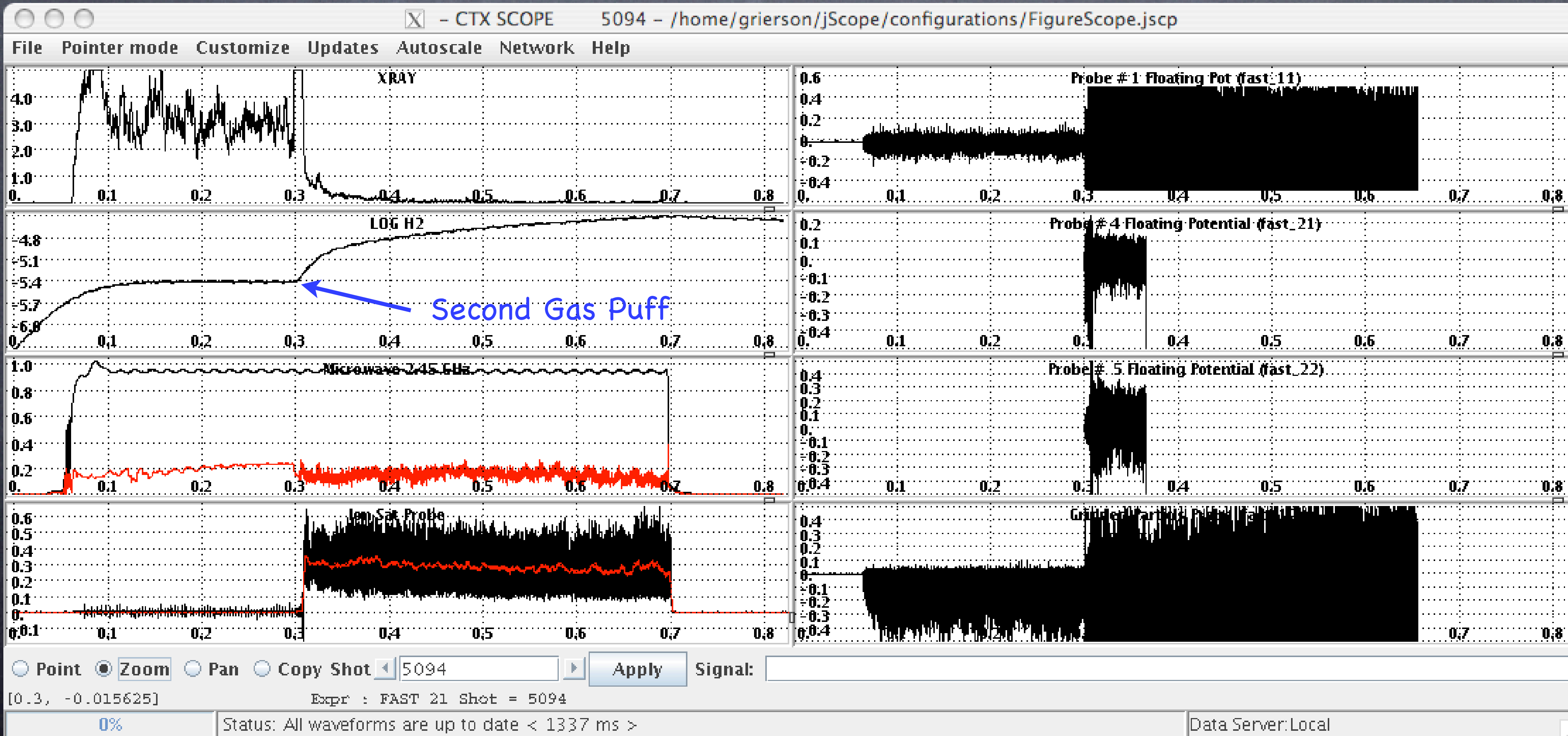
1kW ECRH @2.45GHz

ECRH Resonance at L=27cm

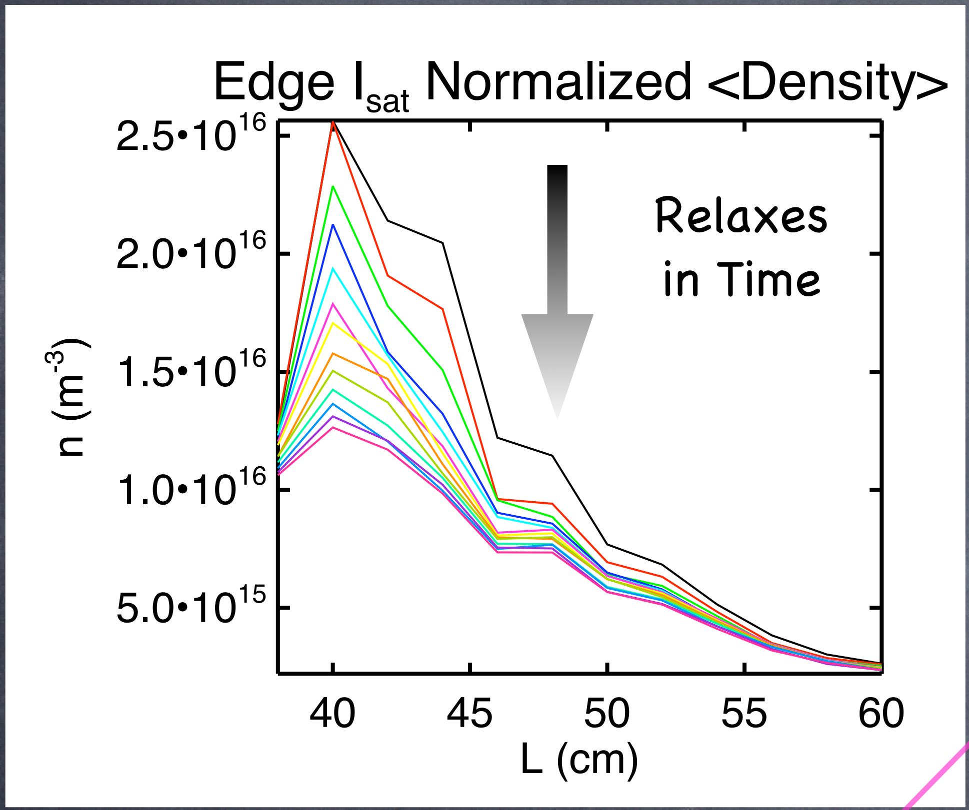
Basic Parameters

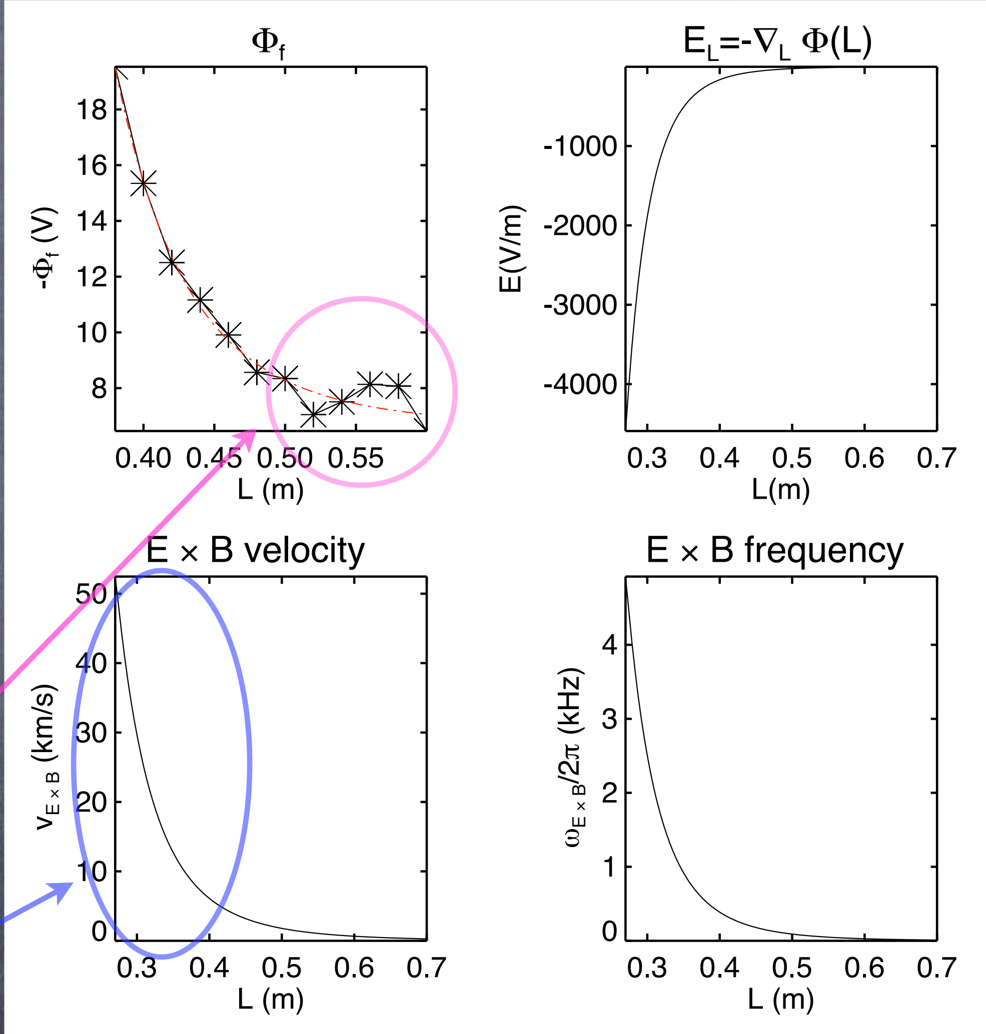
Second gas puff causes:

- massive increase in I_{sat}
- drop of x-rays
- increase of photo-emission



Profiles





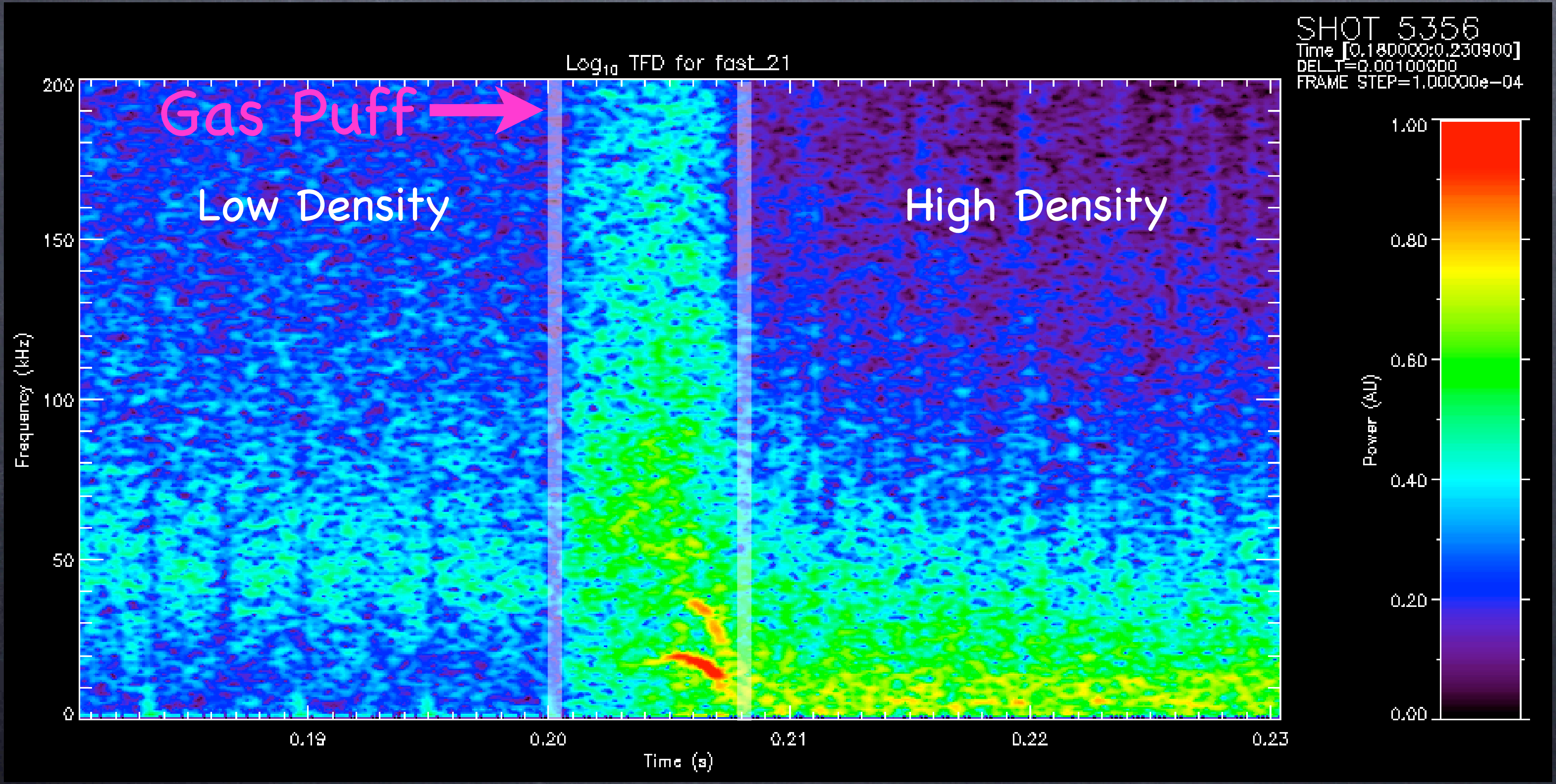
- No Good Profile measurement, turbulent edge.
- Predicts ExB Sheared Azimuthal Flow
- However, measured fluctuations are drift-resonant with ω_d (T_e ~10-12eV)

$$\Phi_f(L) = 0.01L^{-7.5} + 6.6$$

High Density Transition

Large, m=1 Fluctuations

Rigid Rotating ExB (Φ_f~1/L)



Turbulent Time Series

- A turbulent time series displays intermittency.
- A turbulent time series displays a power-law spectra.
- Most data is non-stationary (not strictly periodic), and frequency can change inside of a characteristic period.
- These are problems for Fourier Methods which generally require time series to be:
  - a) Linear.
  - b) Stationary.

Hilbert Spectrum[1]

- Hilbert Transform given by: $$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt'$$
- Form Analytic Function: $$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)}$$
- Instantaneous Frequency: $$\theta(t) = arctan\left[\frac{Y(t)}{X(t)}\right] \quad \omega(t) = \frac{d\theta(t)}{dt}$$
- Instantaneous Amplitude: $$a(t) = \sqrt{X(t)^2 + Y(t)^2}$$
- The phase must be `unwrapped' before differentiating.

IMFs[1]

- In order to apply the Hilbert Transform, the time series must be of the class `Intrinsic Mode Functions'.
- Envelope functions symmetric about the local zero.
- No positive minima or negative maxima.
- Same number of zero crossings as extrama, within one.
- Formed by `sifting the time series'

Sifting Process[1,2]

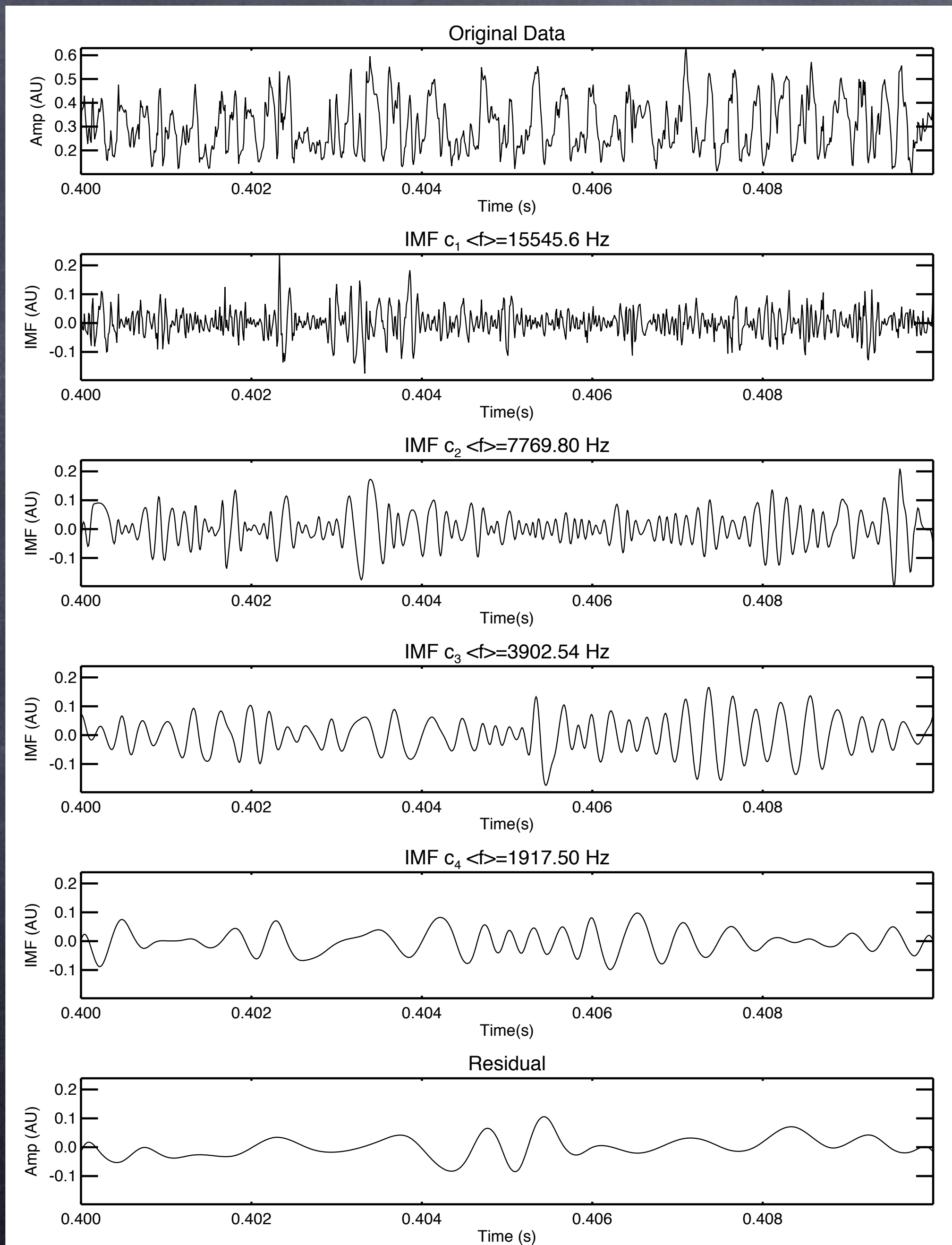
- The time series, $S_0(t)$ is to be sifted into many IMFs.

- One spline is fit to all maxima $S_{max}(t)$, another to all minima $S_{min}(t)$, then the average is taken $m_{11}=(S_{max}(t)+S_{min}(t))/2$.

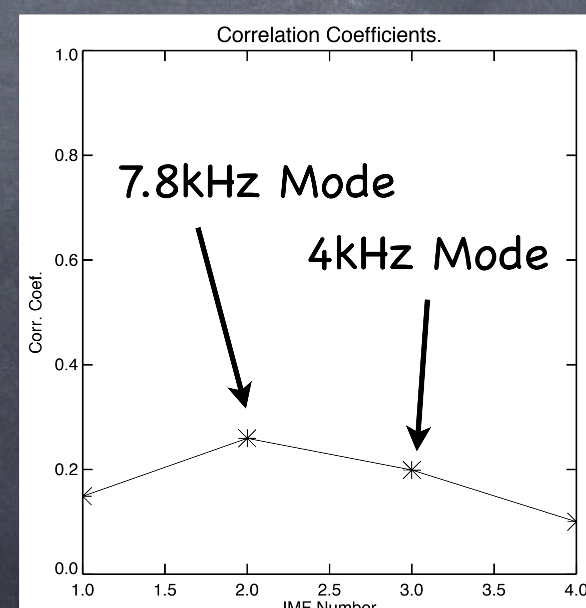
- Subtract spline average from the original signal $S_0(t)-m_{11}(t)=h_{11}(t)$ and repeat until $h_{1k}=h_{1(k-1)}$, where k is the mean spline subtraction iterate. Then $c_1(t)=h_{1k}$, the first IMF.

- $S_0(t)-c_1(t)=S_1(t)$, and repeat process on S_1.

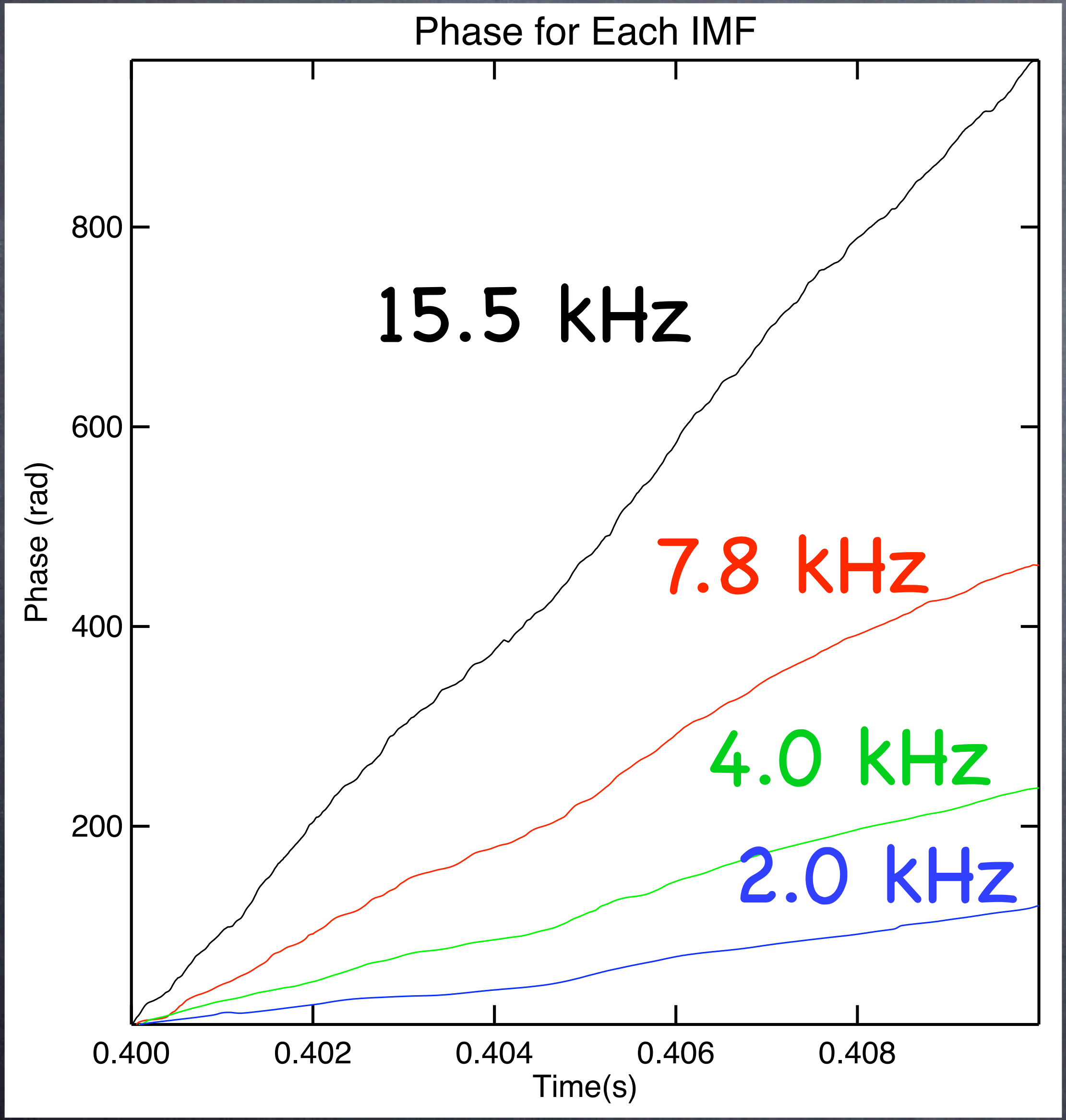
Sifting I_{sat} Data



- The data is sorted into functions with intrinsic time scales that are inherent to the data.
- Each IMF has a frequency which is approximately half the previous IMF

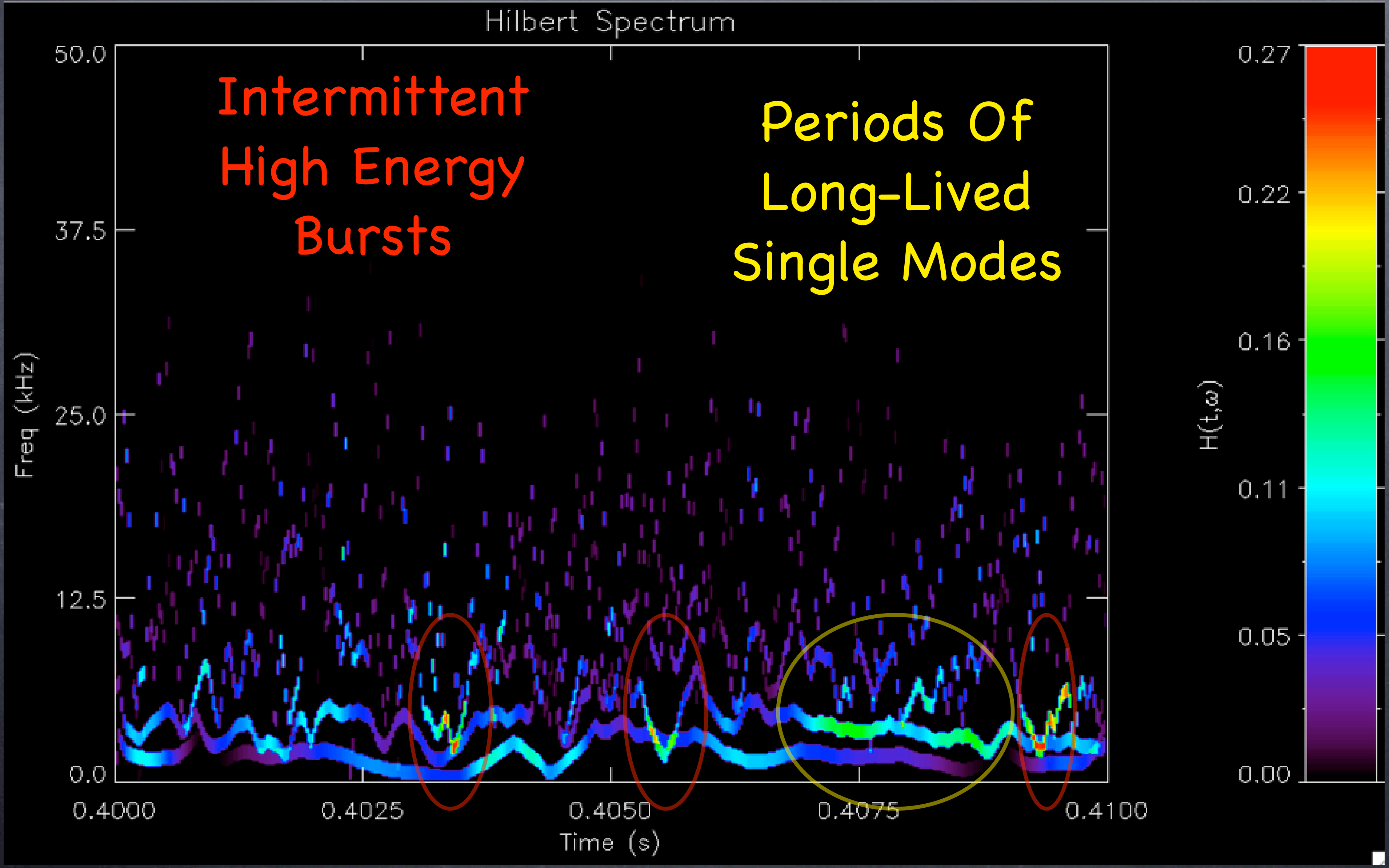


Instantaneous Phase



The instantaneous phase for each IMF. The frequency regimes are well separated. A linear fit gives the average frequency.

Hilbert Spectrum



Statistics

- While the Hilbert spectrum is qualitative, Certain Integral Quantities are quantitative.

Instantaneous Energy

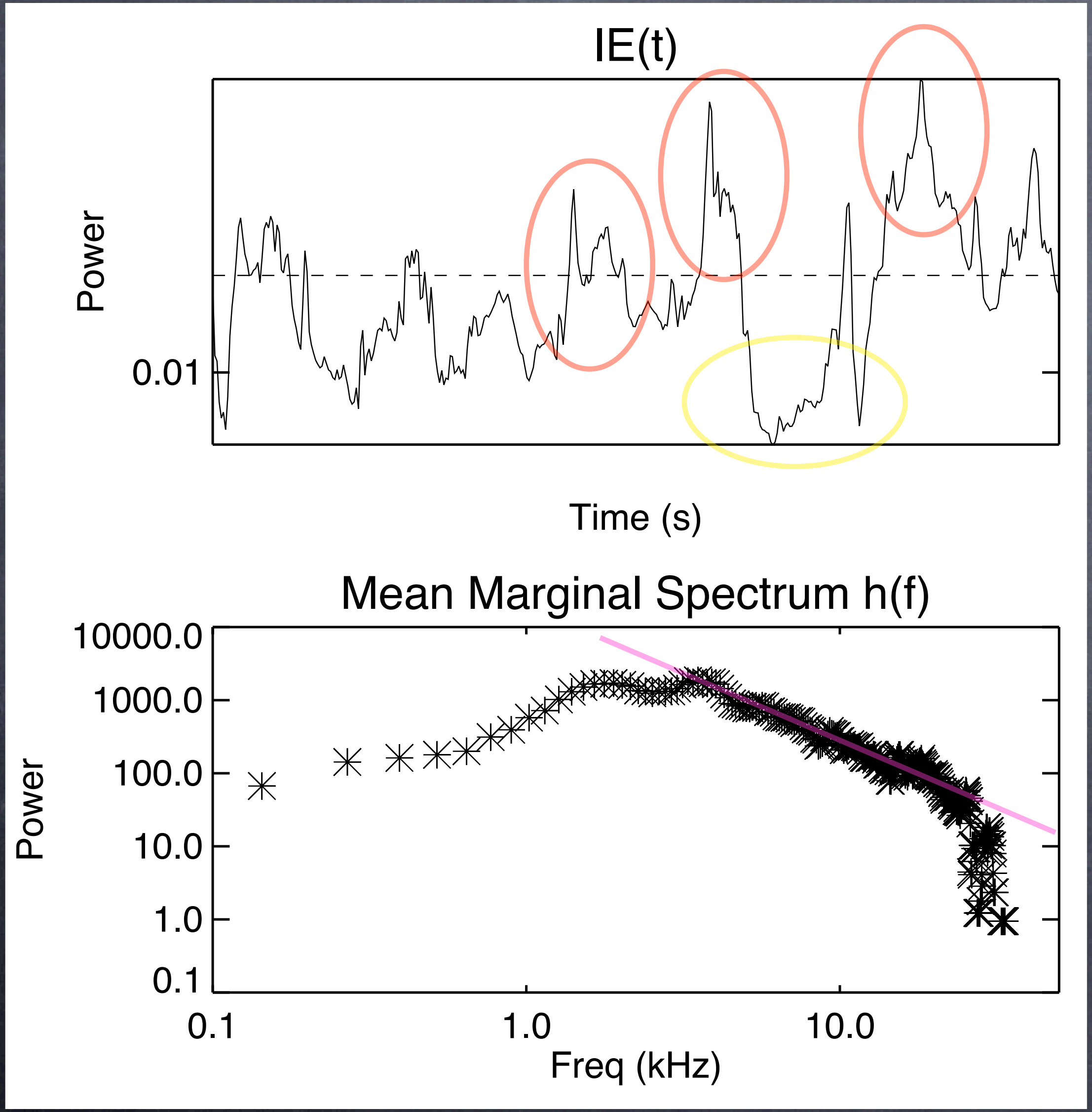
$$IE(t) = \int_0^{\omega_N} H^2(t,\omega)d\omega$$

Mean Marginal Spectrum

Fourier-like spectrum

$$h(\omega) = \frac{1}{T}\int_0^T H(t,\omega)dt$$

IE and Spectrum



- Instantaneous Energy also records the highly energetic, intermittent bursts of low frequency activity
- Spectrum also displays a power-law scaling, similar to FFT.

Summary

- A novel technique has been implemented to study the fluctuations in a turbulent plasma.
- The time series displays intermittent burst of activity, and non-stationary fluctuations.
- The method decomposed a turbulent signal into a few mode functions at intrinsic fluctuation time scales.
- The 4-8kHz frequency range contains most of the power and is most strongly correlated.

[1] Proc. R. Soc. Lond. A (1998) **454**, 903-995
[2] Phys. Plasmas **13**, 082507 (2006)